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RAPID COMMUNICATIONS

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Dynamic fuse model for electromigration failure of polycrystalline metal films

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We introduce a dynamic fuse model for the damage done to a current-carrying polycrystalline metal film by electromigration. For all initial densities of defects p, the mean failure time $\langle T_f \rangle$ is, to an excellent approximation, proportional to the average length of the shortest path across the film in a certain metric. $\langle T_f \rangle$ tends to zero as $(p_c - p)^{4/3}$ as the percolation threshold $p = p_c$ is approached.

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There is currently much interest in random media that are changed irreversibly by an applied field. These so-called "breakdown problems" play an important role in nonequilibrium statistical physics and materials science. "Burnout" of random fuse networks [1–3], dielectric breakdown [2,4–7], the onset of superconductivity in granular superconductors [8], and the fracture of brittle materials [9] have all been studied using breakdown models.

The breakdown models mentioned so far are quasistatic, since failure occurs instantaneously when the applied voltage (or stress) is sufficiently large. A truly kinetic breakdown model was introduced by Sornette and Vanneste [10,11] to describe the failure of fuse networks that burn out due to Joule heating. Their Monte Carlo simulations revealed a rich phenomenology of fracture patterns and the existence of a novel dynamical memory effect [10,11].

When an electrical current passes through a thin metal film, collisions between the conduction electrons and the metal ions lead to drift of the ions. This process is known as electromigration [12]. If there is a divergence in the flux of ions at a point, a void or hillock forms [13]. Voids grow and overlap until conduction ceases and electrical failure is complete. Electromigration can lead to the electrical failure of interconnects in very large scale integrated (VLSI) circuits in relatively short times, reducing the circuit lifetime to an unacceptable level [14]. It is therefore of great technological importance to understand and control electromigration failure of thin films. Electromigration-induced damage in a polycrystalline metal film is an irreversible kinetic process, since the damage cannot be repaired simply by reversing the current.

In this Rapid Communication, we introduce a truly kinetic breakdown model for the damage done to a current-carrying polycrystalline metal thin film by electromigration. In our coarse-grained description, the film in its initial state is represented by a random resistor network in which a fraction p of the bonds are insulators and the remainder are conductors. As current flows through the network, conducting bonds are damaged by electromigration and some eventually fail, becoming insulators. Conduction through the network as a whole ceases at the failure time T_f. For all p, the mean failure time $\langle T_f \rangle$ is, to an excellent approximation, proportional to the average length of the shortest path across the film in a certain metric. This conclusion is supported by our simulations and by analytical work in which we construct exact upper and lower bounds on $\langle T_f \rangle$. We also show that $\langle T_f \rangle$ tends to zero as $(p_c - p)^{4/3}$ as the percolation threshold $p = p_c$ is approached.

We adopt a coarse-grained description of a polycrystalline metal thin film in which the film is represented by an $N \times N$ square grid of sites with a lattice spacing a large compared to the mean crystallite size d. Each nearest-neighbor pair of sites is joined by a conducting bond with a quenched, random conductance. Since $a \gg d$, we may neglect correlations

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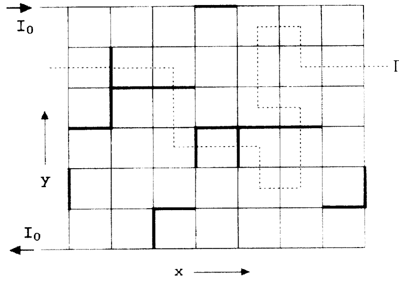

FIG. 1. The initial configuration of a 7×7 network. Conducting bonds are shown in full lines, and insulating bonds are shown with bold full lines. The top and bottom rows (the busbars) are equipotential surfaces. The path Γ on the dual lattice (dashed lines) has "length" $n(\Gamma)=12$. Periodic boundary conditions apply in the x direction, and so the columns to the far left and right coincide.

between the conductances of neighboring bonds. For simplicity, we assume that each bond is either an insulator (with probability p) or a conducting wire with resistance R (with probability $1-p$). All conducting bonds are taken to be identical. Conducting busbars are placed along the top and bottom of the grid and a constant external current I_0 is passed through the network. To minimize finite size effects, we apply periodic boundary conditions in the horizontal direction (Fig. 1).

As current passes through a particular wire in the grid, electromigration occurs and electrical failure eventually takes place. The current I passing through this wire may vary with time since failures elsewhere in the system lead to current redistribution. It is natural to assume that the rate damage is done to the wire at time t is proportional to $|I(t)|$. We further assume that the wire fails irreversibly and becomes an insulator once the damage has reached a given threshold. The lifetime of the wire t_f is then given by $\int_0^{t_f}|I(t)|dt=Q_0$, where Q_0 is a constant with units of charge [15]. The absolute value of the current appears in our failure criterion because the damage rate is independent of the current direction.

Our model is a special case of the more general model of Sornette and Vanneste [10,11]. However, Sornette and Vanneste did not actually study this case, nor did they apply it to electromigration. As we will now show, it is possible to make considerable progress analytically on our model.

Suppose that a single vertical bond is insulating at time $t=0$, and that the remaining bonds are all conducting initially. Our simulations show that the insulating bond nucleates a horizontal "crack" of adjacent broken vertical bonds, and that this crack grows laterally until its tips meet and failure is complete [16]. As an attempt to construct a theory of the failure process when multiple cracks are present, we developed a "Lifshitz-type" theory for our model in which crack-crack interactions are neglected [17]. We were encouraged to do so because a Lifshitz-type theory is quite successful in predicting the failure voltage of a random fuse network [2,3]. Our Lifshitz-type theory is in good agreement with the results of our simulations when $p\ll N^{-1}$, but for larger values of p, an entirely different approach is needed. This is because the interaction and fusion of cracks can be ignored only if the defects are initially very dilute.

We now develop a theory that applies for all p. Let Γ be a path on the dual lattice (Fig. 1). Γ may be any closed self-avoiding path that wraps around the lattice once before closing. For convenience, we will refer to a path of this kind as simply "a path." We now assign a "length" $n(\Gamma)$ to Γ: We let $n(\Gamma)$ be the total number of conducting bonds that cross Γ at time $t=0$. The path with the smallest value of n (the "shortest path") will be denoted Γ_s. We will demonstrate that for any initial configuration C,

$$T_f\leq n(\Gamma_s)t_0, \quad (1)$$

where $t_0\equiv Q_0/I_0$. Most significantly, we argue that the mean failure time

$$\langle T_f\rangle\cong\langle n(\Gamma_s)\rangle t_0. \quad (2)$$

(The angular brackets denote an average over initial configurations C.)

Simulations of our model are extremely time-consuming, since Kirchhoff's equations for the network must be repeatedly solved. However, if we are content with knowing the approximate failure time, we need only compute the shortest path length, and this can be done easily and with great speed [18]. Moreover, much is known about the mean length of the shortest path in certain limits, and this knowledge can be directly applied to our model. Equations (1) and (2) relate the time that a complex dynamical process comes to an end to a simple geometrical quantity, the length of the shortest path through the initial configuration.

To establish the inequality (1), consider an arbitrary path Γ at time t. We walk along the path from left to right, and each time we arrive at a new bond, we assign a sign to the current in the bond. If the current passing through the bond goes from left to right across the path, it is a positive current; otherwise, it is a negative current. We index the conducting bonds that cut Γ at time $t=0$ by the integers $k=1,2,\ldots,n(\Gamma)$. Because charge is conserved, for any time $t<T_f$ we have

$$\sum_{k=1}^{n(\Gamma)} i_k(t)=I_0. \quad (3)$$

Here $i_k(t)$ is the current passing through the kth bond at time t, and has the appropriate sign. Taking the absolute value of Eq. (3) and setting $\Gamma=\Gamma_s$, we obtain

$$\sum_{k=1}^{n(\Gamma_s)} |i_k(t)|\geq I_0. \quad (4)$$

Let $t^*\equiv n(\Gamma_s)t_0$. With this notation, Eq. (1) becomes $T_f\leq t^*$. If the network fails before time t^*, then $T_f<t^*$. Now suppose that the network does not fail before time t^*, so that $T_f\geq t^*$. Consider the situation at time t^*. Integrating Eq. (4) from $t=0$ to t^*, we obtain

$$\sum_{k=1}^{n(\Gamma_s)} \int_0^{t^*}|i_k(t)|dt\geq Q_0 n(\Gamma_s). \quad (5)$$

Since a conducting bond fails once a charge Q_0 has flowed through it, $\int_0^{t^*}|i_k(t)|dt \leq Q_0$ for all k. Equation (5) shows that the only way that this can be so is for $\int_0^{t^*}|i_k(t)|dt$ to be equal to Q_0 for all k. This means that all the bonds that cross Γ_s are insulating at time t^*, and hence $T_f = t^*$. We conclude that $T_f \leq t^* = n(\Gamma_s)t_0$, as claimed.

To obtain a lower bound on T_f, note that once the network has failed, there exists a path Γ_c that is crossed only by insulating bonds [19]. We call such a path a "critical path." T_f is the time that it takes for all conducting bonds that cut Γ_c to break. $n(\Gamma_c)$ conducting bonds cut Γ_c at time $t=0$. If all charges that crossed Γ_c never returned to the region above it, we would simply have $I_0 T_f = Q_0 n(\Gamma_c)$. However, in some cases negative "backtracking" currents cross Γ_c, and we must account for this possibility. Let $Q^-(\Gamma_c)$ be the total amount of charge that returns to the region above Γ_c before time T_f. Explicitly,

$$Q^-(\Gamma_c) \equiv \frac{1}{2}\sum_{k=1}^{n(\Gamma_c)} \int_0^{T_f} [|i_k(t)| - i_k(t)]dt.$$

Each time that a charge returns to the region above Γ_c, it damages conducting bonds twice—once when it backtracks, and again when it returns to the region beneath Γ_c. Thus $I_0 T_f = Q_0 n(\Gamma_c) - 2Q^-(\Gamma_c)$. Since $n(\Gamma_s) \leq n(\Gamma_c)$, we have

$$n(\Gamma_s)t_0 - 2Q^-(\Gamma_c)/I_0 \leq T_f, \tag{6}$$

which is the desired lower bound.

If backtracking across the critical path is negligible on average, $\langle Q^-(\Gamma_c)\rangle$ will be small. Equations (1) and (6) then yield Eq. (2), which relates $\langle T_f\rangle$ to the average length of the shortest path across the initial configuration of the network. Accordingly, we call the approximate theory that is obtained by neglecting backtracking across Γ_c the "shortest path theory."

Is backtracking across Γ_c ever important? Consider the case in which p is small. At early times, backtracking is absent if the broken bonds are remote from one another. Backtracking can occur only if there are several nearby broken bonds, and this occurs with negligible probability. At late times, on the other hand, the current flow is constricted to flow through the few narrow conducting apertures that remain in Γ_c. When the current flow is constricted in this way, it is most unlikely to backtrack. To draw an analogy: If a dam has several small gaps in it, water is unlikely to flow upstream through a hole. We therefore expect that, on average, negative currents across the critical path will be negligibly small for $p \ll p_c$.

Now consider the situation at the percolation threshold $p=p_c$. In most initial configurations C, there is a conducting bond whose removal disconnects the two busbars, a so-called "singly connected bond" (SCB) [20]. Up until time T_f, the SCB carries all of the current. The SCB is the only bond to break, and once it has broken, network failure is complete. In configurations of this kind, the critical path crosses only one conductor (the SCB) and there is never any backtracking. If there is no SCB in C, it is practically certain that the simultaneous removal of two conducting bonds is sufficient to interrupt the flow of current [20]. These bonds

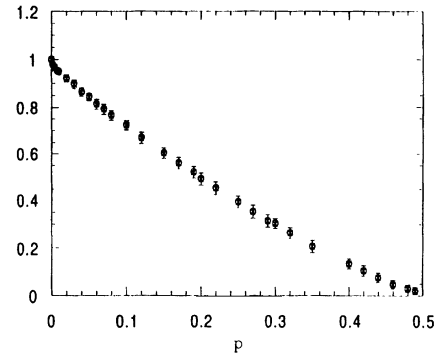

FIG. 2. The dimensionless failure time $(j_0 a/Q_0)\langle T_f\rangle$ (crosses with error bars) and $\langle n(\Gamma_s)\rangle/N$ (open circles) are plotted vs p for a 64×64 network.

are called "doubly connected bonds." In all but rather unusual initial configurations, the critical path crosses only two conductors, the doubly connected bonds. If this is the case, there is never any backtracking across the critical path. We conclude that at the percolation threshold, $\langle Q^-(\Gamma_c)\rangle$ is small. It is also possible to argue that backtracking is negligible for p close to but less than p_c [17]. In sum, we expect Eq. (2) to be a good approximation if p is small or is close to p_c.

To test the shortest path theory, we performed simulations of our model. Instead of solving Kirchhoff's equations directly, the Green's function formulation for the resistor network was solved using the conjugate gradient method [21]. T_f was determined for grids with $N=64$ for a range of p values between 0 and $p_c = \frac{1}{2}$. For each value of p, an average over 50 configurations was made. The current density $j_0 = I_0/(Na)$ and the charge Q_0 had the same values in all of the simulations.

We also computed the shortest path lengths for the same set of initial configurations that were used in our simulations of the electromigration process. These lengths were averaged to give an estimate of $\langle n(\Gamma_s)\rangle$. For a given initial configuration, $n(\Gamma_s)$ was determined using a straightforward modification of the "burning" algorithm [18].

The computed values of $\langle n(\Gamma_s)\rangle/N$ and $(j_0 a/Q_0)\langle T_f\rangle$ are very close to one another for all values of p (Fig. 2). Our results therefore provide strong support to our claim that Eq. (2) is a good approximation for small p and for p close to p_c. Indeed, the shortest path theory works very well throughout the entire range of p values.

Much is known about the behavior of $\langle n(\Gamma_s)\rangle$ [22]. It has been proven that as the size of the network $L \equiv Na$ tends to infinity, $\langle n(\Gamma_s)\rangle \sim \mu(p)L$. The first passage time constant $\mu(p)$ is positive for $p<p_c$ and is zero for $p>p_c$. As the percolation threshold p_c is approached from below, $\mu(p)$ tends to zero as $\mu(p) \sim (p_c - p)^\nu$. The exponent ν is exactly $\frac{4}{3}$ in two dimensions [23]. At the percolation threshold, $\langle n(\Gamma_s)\rangle \sim k \ln L$ as $L \rightarrow \infty$, where k is a constant.

Let us now consider the predictions of the shortest path theory. For a given $p<p_c$, the mean failure time grows as $\langle T_f\rangle \sim \mu(p)t_0 L$ as $L\rightarrow\infty$. The failure time tends to the constant $\mu(p)Q_0/j_0$ if the current density j_0 is held fixed as L

grows large. If p is not greater than p_c but is close to it, we may use the scaling hypothesis to obtain

$$\langle T_f\rangle \sim \begin{cases} t_0 \ln L & \text{for } a \ll L \ll \xi; \\ (p_c-p)^{4/3} t_0 L & \text{for } L \gg \xi. \end{cases}$$

Naturally, $\langle T_f\rangle$ is zero for $p > p_c$.

In our simulations, we observed that the critical path Γ_c tends to be close to the shortest path Γ_s, and that large portions of these two paths often coincide. In most instances, $n(\Gamma_c)$ is equal to $n(\Gamma_s)$ or is only slightly larger than it. This can be shown to be the case using the shortest path theory. Recall that $T_f = n(\Gamma_c)t_0 - 2Q^-(\Gamma_c)/I_0$. Combining this with Eq. (1), we obtain $n(\Gamma_c) \leq n(\Gamma_s) + 2Q^-(\Gamma_c)/Q_0$. On the other hand, $n(\Gamma_s) \leq n(\Gamma_c)$. Since $\langle Q^-(\Gamma_c)/Q_0\rangle$ is negligible, we have $\langle n(\Gamma_c)\rangle \cong \langle n(\Gamma_s)\rangle$.

Ideas reminiscent of our shortest path theory have been introduced in the theory of quasistatic dielectric breakdown [6,7,24]. In their Monte Carlo simulations of a continuum model for dielectric breakdown in metal-loaded dielectrics, Gyure and Beale assigned each path P that spanned the system from busbar to busbar a "gap" $x(P)$ equal to the length of path that lies in the dielectric [7]. Gyure and Beale noted that the path with the smallest gap tends to be close to the actual path that led to breakdown. Their simulations also suggest that the breakdown field is approximately proportional to the minimum gap. At present, there is no analytical work to support these observations.

In this Rapid Communication, we have concentrated on the failure of square thin films. However, the shortest path theory can also be used to account for experimental results on the lifetime of long, narrow wires, which is an issue of great importance in microelectronics [17]. In the future, we intend to simulate networks with more general types of disorder than percolative disorder. The shortest path theory is readily generalized to more general types of disorder in the bond resistances, and to problems in which Q_0 is a quenched random variable as well [17]. We are also investigating the possibility that variants of the shortest path theory can be applied to other models of breakdown in random media.

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